### A Survey on Spatial Association Rule Mining Technique and Algorithms for mining spatial data Banalata Sarangi, (Prof.) Dr. Laxman Sahoo

Abstract – spatial association rule mining is an important technique of spatial data mining. Mining spatial association rule is one of the most important branches in the field of spatial data, spatial data mining can extract the spatial patterns and characteristics, general relations of spatial and non spatial data and other data features in common that hidden in spatial database. This paper describes and explains various spatial association rule mining algorithms and methods. Here we try to give a detailed survey of the existing spatial association rule mining technique based on Buffer analysis, Maximum frequent item sets based on Boolean matrix, concept lattice.

Index Terms— Data mining, Spatial mining, spatial association rule, and spatial data mining.

#### 1 INTRODUCTION

Data mining in general is searching for hidden and interesting patterns that may exist in generic data. Spatial data mining in particular is discovering the interesting relationships and characteristics that may exist implicitly in spatial data. Association rules analysis was only used to find out frequently appearing transactions in relational database based on apriori, but with the development of geographic information technology, people have not satisfied with the traditional data mining.

#### 2 SPATIAL ASSOCIATION RULES STUDY BASED ON **BUFFER ANALYSIS**

This method finds out longest groups in which object influents each other.

#### 2.1 Influence area and buffer area.

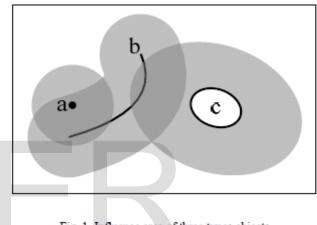
All spatial objects, no matter belongs to any type, will influent surrounding objects. In this paper, the influence radius area also called buffer area. It can be said that object have some influence or contact on other objects which locate inside the influence radius area.

#### Theorem 1:

If a's influence radius is ra, and a's buffer area intersects with b, then it can be said that a influence on b.

Exists (b Interact (Buffer (a, ra)) = true

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#### Fig. 1. Influence area of three types objects.

### 2.2 Support and Confidence

Formula 1:

Spatial objects have bigger Support degree, means they have bigger ratio to all spatial object, and means they are more important.

Formula 2:

Some spatial objects like broadcast station have very large range so frequency will not take more important to calculate support. So

#### Confidence-

If X and Y, two type spatial objects, and their influence radiuses are rx and rr. Function Count(X) returns the X type objects number in spatial area, and function Count(X\*(Y©ry)) returns the X type objects number which inside Y influence area.

#### Formula 3

**IJSER © 2013** http://www.iiser.org Confidence( $Y \Rightarrow X$ ) =  $\frac{\text{Lowerth [A = [y \Rightarrow y]]}}{1}$ 



#### Theorem-1

When the confidence of X influencing y and the confidence of y influencing X are all bigger than a threshold c%.So X and y are appearing frequently.

Min (Confidence(X => y) , Confidence(Y => X)) > c% C% = Confidence threshold.

#### 2.3 Basic Algorithm steps of finding Association rules:

Assume there are j kinds of spatial objects in the spatial area, AI, A2, ..., Ai' the main steps of the basic algorithm to find out the longest association groups are as following:

Step 1: Based on Formula 1 or Formula 2, pick out these types of objects which satisfy the Support Threshold s%.

Step 2: According to Formula 3, calculate Confidences of all two-tuple groups.

Step 3: Set n=2. Then, according Theorem 2 comparing the Confidence threshold c%, pick out all groups with n- ary association. If these groups don't exist, the algorithm will exit directly.

Step 4: Set n=n+ I, if n>j, then jump to step 6. Otherwise, process previous step results according to Theorem 3, pick out all groups with n-ary association.

Step 5: If n-ary association groups don't exist, then jump to step 6. Otherwise, store all n-ary association groups, and jump to step 4.

Step 6: Output all groups with n-ary association

#### 2.4 Advantages of the algorithm

This algorithm reduces human intervention and suits for all shapes of spatial object.

#### 3 SPATIAL ASSOCIATION RULE BASED ON BINARY MIN ING ALGORITHM

In areas like mobile computing there are many spatial data correlative with locations, which are very important for mobile inteligent system to extract spatial association among locations that can provide potential and useful information formobile client.this paper proposes an approach of extracting spatial association rules based on binary mining algorithm.

#### 3.1 THE ALGORITHM OF ASSOCIATION RULES MINING BASED ON BINARY

Let  $I = \{i1, i2...im\}$  be a set of items, if  $ik \in I$  let  $T = \{i1 \land i2 \land ... \land$ im} ( $Tk \subseteq I$ ) be a subset of items, named a transactions. For example, let  $Tk = \{i1, i2, i3\}$  be a subset of items, called a transaction. And then let  $D = \{T1, T2...Tn\}$ , let  $Tk \subseteq I$ , (k=1...n) be a set of transactions, called Transaction Database (TD).

BT-Binary Transaction (BT), a transaction is expressed as bina-

ry, binary transaction of transaction *T* is expressed as BT = (b1 b2...bm),  $bk \in [0, 1]$ , k=1...m. If  $ik \in Ti$ , and then bk=1, otherwise bk=0, the order of binary digits are as same as items which have been fixed. Example, Let  $I=\{1,2,3,4,5\}$  be a set of items, if a transaction is expressed as  $Ti=\{2,3,5\}$ , and then BTi=(01101).

DT-Digital Transaction (*DT*), which is the integer, the value of which would be obtained by turning binary of transaction into algorism. Example, If *BT*=01101, and then *DT*=13.

FDT- Frequent Digital Transaction (*FDT*), which is a digital transaction, the support of which excess minimal Support given by users.

CDTS- Candidate Digital Transaction Section (*CDTS*), which is an integral section from 3 to max, each power of 2, does not belong to *CDTS*.

Max=*BTi1* v *BTi2*...v *BT*jk, each *BT* only expresses a kind of item, their support excess minimal support given.

#### 3.2 The Process of Association Rules Mining Based on Binary

Firstly, we define some signs as follows:

*DB*: *DB* is used to save digital transaction.

*D*: data-domain of *D* contain "value" and "count", "value" is used to save digital transaction and "count" is used to save the number of same digital transaction.

*FDT*: data-domain of *FDT* contains "value" and "count", "value" is used to save digital transaction and "count" is used to save support of digital transaction, which excess minimal support given by users.

*NFDT*: data-domain of *NFDT* only contains "value" to save digital transaction, the support of which is under minimal support given, and then go on doing:

**Step 1:** Data Transformations. Transaction would be transformed into digital transaction from traditional database and then digital transactions would be saved in *D* on descending by digital transaction.

**Step 2**: Creating candidate digital transaction section (*CDTS*).Frequent digital transactions gained by scanning *D*, are used to create, which only express a kind of item.

**Step 3:** Forming sets of Frequent Digital Transaction (*FDT*).All frequent digital transactions are searched from *CDTS* to save in *FDT*.

**Step 4:** Creating Digital Association Rules. Digital association rules are created from *FDT* when their confidence Excess given min-confidence.

#### 3.3 The Algorithm of Association Rules Mining

Let [3, max] be a *CDTS*, and there are *N* digital transactions saved in *D* on descending by digital transaction, where data aren't repeated, and the algorithm which generates digital association rules after search frequent digital transaction, is ex-

#### pressed as follows:

(1) While  $(DT \in [3, max])$  { (2) If (all  $NFDT_i \notin DT$ ) { (3) While  $(DT_Di.value\&\&i_N)$  { (4) If  $(DT \subseteq Di.value)$ (5) s count+= Di.count; (6) i++;//computing support of DT (7) If (s\_count/N\_support) { (8) Delete all *FDT*k (*FDT*k $\subset$ *DT*) from *FDT*; (9) Write *DT* and s\_count to *FDT*; (10) }//checking frequent digital transaction (11) Else (12) Write *DT* to *NFDT*;} (13) *DT*++; (14) }//searching all frequent digital transaction (15) For (all  $DT \in FDT$ ) { (16) *DT* =*FDT*.value; (17) s\_count=*FDT*.count; (18) Create\_Rules (*DT*, s\_count); (19) }//generating association rules Create Rules (*DT*, s count); (1) While  $(sub \in [1, DT] \&\& sub \subset DT)$  { (2) For (all  $Di \in D$ ) { (3) If  $(sub \subseteq Di.value)$  c\_count+= Di.count; (4) *i*++;} //computing support of sub (5) If (s\_count/c\_count\_confidence) (6) Display sub *DT*& (~sub); (7) sub++ ;}

#### 3.4 Process of generating spatial association rules

The process is expressed as follows:

**Step 1:** Digital association rules are transformed into binary, if digital "1" exists in some binary bits, transaction-item (*i*) related to each bit, and then comprehensible association rules are expressed as  $\{i1,i2\}_{i4}$ .

**Step 2**: Item (*ij*) of comprehensible association rules would be renewed into spatial predicate close\_to (T, O), and then the spatial association rules are expressed as follows:

close\_to (T, O1) **n**close\_to (T, O2) \_close\_to (T, O4) **Step 3**: The normal spatial association rules are expressed as follows:

is\_a (X, T) nclose\_to(X, O1) nclose\_to(X, O2) \_close\_to(X, O4) [30%, 80%]

Let *X* is an objective which is hotel, so above rule is explained as follows: When percent 80 of hotel are close to *O*1 (mall) and *O*2 (traffic-service), they are also close to *O*4 (bank), there are percent 30 of data accord with the rule in transaction database.

#### 3.5 Advantages of the algorithm

The efficiency of algorithm based on binary is faster and more

efficient than apriori, and so it made the efficiency of mobile device be improved.

## 4 SPATIAL ASSOCIATION RULE BASED ON BOOLEAN MATRIX

## 4.1. Creating a Boolean matrix according to frequent Length-1 item sets.

All the frequent length-1 itemsets will be generated from transaction database using the Apriori algorithm when transaction database is scanned first time and for each frequent length-1 itemset, all the IDs of transaction records containing it need to be taken note in one array. Then the corresponding Boolean array with the length being the number of the transaction records in database will be created for each frequent length-1 itemset. In each array, there are only two values, '0' and '1'. If transaction record contains frequent length-1 itemset, the value is 1 in the corresponding Boolean array, vice versa.

At last, a Boolean matrix will be constructed according to all the Boolean arrays of frequent length-1 itemsets.

1) *Definition 1:* The corresponding Boolean array of each frequent length-1 itemset Im[N] is {*BT1, BT2, ..., BTn*} (1

 $n \Box N$ ), where *Im* is the *mth* frequent length-1 itemset; *N* is the number of transaction records in database; *Tn* is ID of the *nth* Transaction record respectively; and *BTn*'s value is 0 or 1 only. 2) *Definition 2:* The Boolean matrix of frequent length-1 itemsets *IM*\**N* is {*I*1 [*N*], *I*2 [*N*], ..., *Im*[*N*]} ( $1 \Box m \Box M$ ), where *Im*[*N*] is the Boolean array with *N* dimensions of the *mth* frequent length-1 itemset; *M* is the number of frequent length-1

item sets.

3) The pseudo codes of the first part (Fig. 1):

Input: Transaction Database D,

Minimum support min\_sup

Output: Frequent length-1 itemsets L,

#### Begin

Find all the frequent length-1 itemsets  $L_i$  from D with the Apriori if  $L_i$  is not null

for each  $I_m$  in  $L_i$ for each I in Dif t.contains $(I_m)$   $I_m|t|=1$ else  $I_m|t|=0$ return  $I_m|N|$ end for end for end if  $I_{M+N} = \{I_i|N|, J_2|N|, ..., I_m|N|\}$ End

Figure 2. The pseudo codes of the first part

## 4.2 Extracting Maximum Frequent Itemsets from Boolean Matrix

Each column in the Boolean matrix represents one transaction record. Value 0 in the column means the corresponding transaction record contains the corresponding frequent length-1 itemset, vice versa. Therefore, the number of value 1 in each column indicates the corresponding transaction record contains the number of frequent length-1 itemsets together. If there is the number of transaction records with the same number of value 1 being larger than the minimum support, the number of value 1 may be the size of maximum frequent itemset, vice versa. As a result, a set of values in which each one may be maximum frequent itemset's length will be obtained. Then according to each of the values in descending order, a series of candidate itemsets will be generated from frequent length-1 itemsets and the support of each candidate itemset could be calculated according to the Boolean matrix of frequent length-1 itemsets. If the support of each candidate itemset is larger than the minimum support, the candidate itemset is frequent, vice versa. At last, if the maximum frequent itemsets generated from the set of candidate itemsets are not empty, the size of candidate itemset is required, that is length of maximum frequent itemset. Otherwise, it is necessary to continue the previous operation to check the next value until maximum frequent itemsets are not empty. If all the maximum frequent itemsets are empty, the maximum length of frequent itemset is one.

- Definition 3: Max[n] is an array used for storing some values of which each may be the length of maximum frequent itemset, where n is the size of Max[n].
- Definition 4: The set of candidate itemsets of maximum frequent itemsets C is {*IM*1, *IM*2, ..., *IMn*}, therefore, the Corresponding Boolean matrix *CMn\*N* is {*IM*1[*N*], *IM*2[*N*], ..., *IMn*[*N*]}, where *IMn* is candidate itemset.
- 3) Definition 5: The support of candidate itemset C, Support(C) = IM1[N] And IM2[N] And ... IMn[N]. Fig. 3 shows the example of the logical Boolean operator "And" between the Boolean arrays of candidate item sets, where "And" is the logical Boolean operator, if there exists value 0, then the calculation will be 0.
- 4) The pseudo codes of the second part (Fig. 3):

```
Input: The Boolean matrix of frequent length-1 itemsets I_{u+v},

Minimum support min\_sup,

Frequent length-1 itemsets L_v

Output: Maximum frequent itemsets L_v

Begin

for each column in the Boolean matrix I_{u+v}

calculate the number of value 1 in the current column

end for

return Max[n]

Sort(Max[n])

for each one in the Max[n]

calculate the number of the columns with the same number of value 1

if the number>min\_sup

generate maximum length candidate itemsets from L_i
```

for each *itemset* in candidate itemsets calculate Support(*itemset*)

if Support(itemset)>min\_sup

itemset is frequent

end if

end for

end if

if Maximum frequent itemsets is not null

break;

end if

end for End

Figure 3. The pseudo codes of the second part

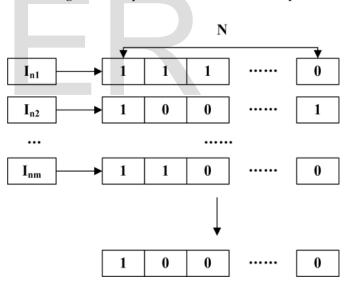


Figure 3. The logical Boolean operator of the Boolean arrays of the set of candidate itemsets

#### 4.3 Generating All the Frequent Itemsets from Maximum Frequent Itemsets

All the frequent itemsets could be extracted from all the maximum frequent itemsets according to the nonempty subsets of frequent itemsets being still frequent. And the support of each frequent itemset could be calculated by Definition 5. At last, all the strong association rules can be mined from all the frequent

#### itemsets.

#### 4.4 Advantages of the algorithm

The run time of this algorithm was much less than the apriori. It reduces times of scanning transaction database and decreased the number of the set of candidate itemset according to the comparision of the principles between two algorithms, so is superior to apriori.

## 5 SPATIAL ASSOCIATION RULE MIKNING BASED ON CONCEPT LATTICE

#### 5.1 Definition of spatial concept lattice

Concept lattice is the core data structure in formal concept analysis. Each concept is expressed by one node in concept lattice. Formal concept is the concept which is expressed formally. Every formal concept includes two parts extent and intent[4].

- Extent –examples contents by concept is the set of all objects included by concept, is the set off objects included by concept.
- Intent-he description of concept is the common character of all examples contented by the concept.

Formal concept analysis expresses extent and intent by the formalized mathematical language.

Both generalization and the specialization relations among the objects can be described by the structure of concept lattice. Concept lattice are visualized by Hasse diagrams.

#### Process of building concept form database.

The formal context is a triple T= (O, D, R).

O is the set of objects under consideration,

D is the set of their descriptors (attributes),

R is a binary relation between O and D.

There is a unique lattice structure lattice L, Galios lattice or concept lattice (Wille R 1982), corresponding to the formal concept (T (O, D, R)). And it is generalized by a partially ordered set.

Correspondingly, besides the set of descriptors (attributes),

Each node of lattice L is a pair (Y, X) as concept where Y is the extension and X is the intension.

Y is the set of objects of power set P (O);

 $X \square P(D)$  is the set of all common attributes of objects in Y. .Each pair (Y, X)  $\square P(O) \times P(D)$  is a complete pair to relation R, that is, it satisfies:

 $Y= \{y \square \ O \mid x \square X, x \ R \ y\}$  $X= \{x \square \ D \mid y \square Y, x \ R \ y\}$ 

CS (K) denotes the set of all formal concepts of K . The foremost structure of CS(K) is built by subconcept-superconcept relation (generalization - specialization relation, or prosequence - subsequence relation ). It defines: if O1 $\leq$ O2, the formal concept (O1,D1) is the subconcept of the formal concept (O2,D2). It is denoted as: (O1,D1)  $\leq$  (O2,D2)

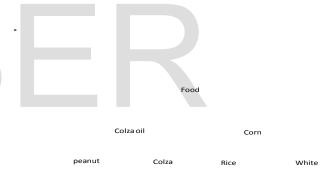
Each node in concept lattice is a formal concept. Their relations can be described as relation of concept.

In concept lattice, there are two different nodes C1 = (O1, D1) and C2 = (O2, D2). if C1is the sub-concept of C2, and there is not any other node C3, which satisfies:C1  $\leq$ C2  $\leq$ C3,Then c1=child node of c2

Then, C1 is the child node (direct subsequence) of C2, andC2 is the father node (direct prosequence) of C1.This paper denotes intension and extension of node C by Intension(C) and Extension(C).

#### 5.2. GENERALIZATION OF SPATIAL RELATIONSHIP-

There are two methods by which to build spatial concept lattice. One is to build straight from the spatial data; the other is to generalize spatial relationships into attribute relationships.Generalization is controlled by non-spatial data. This strategy is to generalize non-spatial data primarily and then adjust the attribute of spatial data



#### **Figure 4**

When non spatial attributes are generalized to the higher concept layer, spatial information should be united, spliced and so on.The main producing area of rice and wheat united when the main producing area of corn are presented. Generalize the spatial relationship to the spatial concept layer and build the spatial concept lattice based unit.

The table below is acquired after generalizing a spatial database according to Fig. 1

A: Be adjacent to a river or not

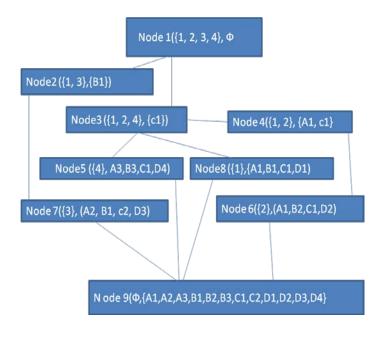
- B: The yield of peanuts
- C: The yield of colza's
- D: The yield of rice

Suppose there's a generalized spatial database as below

#### TABLE I. TABLE FOR BUILDING HASSE GRAPH

TID	А	В	С	D
1	A1	B1	C1	D1
2	A1	B2	C1	D2
3	A2	B1	C2	D3
4	A3	B3	C1	D4

A1-Adjacent to river A2-contiguous to river A3-Far away from river B1-High yield of rice B2-middling yield of rice B3-low yield of rice C1-high yield of corn C2-low yield of corn D1-High yield of of wheat D2-Relatively high yield of of wheat D3-middling yield of of wheat D4-Low yield of of wheat



#### Figure-5 Hasse Diagram

#### 5.3 ASSOCIATION RULE MINING ALGORITHM BASED ON SPATIAL CONCEPT LATTICE

**Theorem 1**-If node H=(X, X') in the lattice has only one parent node M = (Y, Y'), then any rules foreside created from H could only be a single descriptor and for  $P \square \{X'-Y'\}$  there exists a rule P=>X'-p without redundancy.

**Theorem 2-** If node H=(X, X') in the lattice has parent nodes with the number of d, which are M1

(Y1,Y'1),M2(Y2,Y'2),...Md(Yd,Y'd),then for any descriptor p  $X'-(Y'1\Box Y'2\Box...\Box Y'd)$ ,there exist a rule P=>X'-p.The total number of rules with a single descriptor is  $\Box X'\Box-\Box Y'1\Box Y'2\Box...\Box Y'd\Box$ 

**Theorem 3-** If node H= (X, X') in the lattice has two parent nodes M = (Y1,Y'1) and M2 = (Y2,Y'2), then for  $\forall p1 \Box \{Y'1 \Box Y'1 \Box Y'2 \& \forall p2 \Box \{Y'2 \cdot Y'1 \Box Y'2\}$ , there exist a rule p1 p2 => X'-p1 p2, and the total number of rules with two descriptor as its foreside is  $\Box Y'1 \cdot Y'1 \Box Y'2 \Box * \Box Y'2 \cdot Y'1 \cap Y'2 \Box$ ;

#### ALGORITHM

Enter- a lattice L;

Output- rule set R and rules set Rules [H] without redundancy of each node H;

FUNCTION Create Rules

For Node (H =(X,  $X \Box$ ), k)

/ \*To create rules with foreside which are descriptors with the number of k for node H \*/

BEGIN

IF k=1 // If the node has only one parent node P(Y, Y $\square$ ) THEN

According to Theorem 1 that Create rules with only one descriptor as their foreside based on Theorem 1

for ( int i=0; i<= the amount of item sets in parent nodes ;
i++;)</pre>

 $Rules[H] = Rules[H] \Box (yi=>X \Box - yi)$ 

ENDFOR

else

if the amount of item sets in X' < k then return else

Create rules with foreside which are descriptors with the number of k based on Theorem 2&3

for ( int j=0;j<=the amount of parent nodes of H; j++)

for ( int i=0; i<= the amount of item sets in parent nodes :i++; )

Rules [H] = Rules [H]  $\Box$ (y1y2 ...yi =>X $\Box$ - y1y2 ...yi )

ENDFOR

ENDFOR

END

BEGIN

PROCEDURE Create R (Lattice L, Rules R) //Mine association rules from concept lattice

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Initialize rule set  $R = \Phi$ 

Go through the chain table L for storing the concept lattice, and mine association rules for each node FOR (int i=0; i<=the amount of nodes in the chain table; i++) R =R □Create Rules for Node (H) ENDFOR

END

#### 5.4 Advantages of the algorithm

The algorithm can be emlimented on variety of data set having different format and is proved to be faster than apriori algorithm.

#### 6. CONCLUSION

This Paper gives a detailed survey of four spatial association rule mining algorithm which uses the technique like buffer analysis, boolean matrix, binary mining, concept lattice. This paper summarise different techniques which helps the reasearcher in there work.

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